Numerical Examples: Radioactivity

1. At $t = 0$, N_o number of radioactive nuclei A start decaying into B with a decay constant λ_a. The daughter nuclei B decay into nuclei C with a decay constant λ_b. What is the number of nuclei B at a small time t? **[JAM-2021]** Ans.: $N_o\lambda_a t$

2. The radioactive nuclei K^{40} decay to Ar^{40} with a half-life of 1.25×10^9 years. The K^{40}/Ar^{40} isotopic ratio for a particular rock is found to be 50. The age of the rock is $m \times 10^7$ years. What is the value of m? (Rounded off to 2 decimal places) ? **[JAM-2020]** Ans.: 3.57

3. If $\bar{p}$ be the mean momentum of a nucleon in a nucleus of mass number A and charge number Z then which of the following is correct? **[JAM-2019]**
(a) $\bar{p} \propto A^{1/3}$ (b) $\bar{p} \propto Z^{1/3}$ (c) $\bar{p} \propto A^{-1/3}$ (d) $\bar{p} \propto (AZ)^{-2/3}$ Ans.: (c)

4. In a rock sample the ratio of U^{238}: U^{235} is found to be 137.8: 1. If it is assumed that at the birth of Earth the two isotopes occurred in equal abundance in rocks, then determine the upper limit of the age of the planet. Given, the half-lives of U^{238} and U^{235} are respectively 4.5×10^9 years and 7.1×10^8 years. Ans.: 6×10^9 years

5. Find the activity of K^{40}, in Ci units, in the human body assuming that the average weight of a person is 60 kg and 0.4% of the body weight is potassium. The abundance of K^{40} in natural potassium is 0.01% and its half-life is 1.3×10^9 years. Ans.: 1.64 $\mu\, Ci$.

6. A proton is confined within a nucleus of size $10^{-13}\ cm$. If $h = 6.626 \times 10^{-34}\ J\ s$ and $m_p = 1.672\ \times 10^{-27}\ kg$, then find the uncertainty in the velocity of the proton rounded off to two decimal places. **[JAM-2019]** Ans.: $\approx 3.2\ \times\ 10^7\ ms^{-1}$

7. If a proton to capture an electron to form a neutron and a neutrino (assumed massless), the electron must have some minimum energy. For such an electron what will be the de Broglie wavelength if the masses of neutron, proton and electron are respectively $m_n = 939.57\ MeV, m_p = 938.27\ MeV$ and $m_e = 9.1 \times 10^{-3}\ \ kg$.
[JAM-2017] Ans.: 1.08 pm

8. An ancient piece of wooden furniture shows C^{14} activity which is $\frac{4}{5}$ of the activity of a living tree from which such furniture are made. If half life of C^{14} is 5700 years, then find the age of the old furniture correct to first decimal place. Ans.: 1.8×10^3 years.

9. A radionuclide X is produced in a reactor at a constant rate η. If λ be the decay constant of the radionuclide, then find the number of the nuclei at an instant t after the production of X starts. Ans.: $\frac{\eta}{\lambda}\left(1 - e^{-\lambda t}\right)$

10. Find the isotope formed when $^{226}_{88}Ra$ undergoes five alpha decays and four beta decays. Ans.: $^{206}_{82}Pb$.

11. A beam of monoenergetic gamma rays is incident on an aluminium sheet of thickness 10 cm. The sheet reduces the intensity of the beam to 21 percent of the original. Calculate the linear and the mass absorption co-efficients of aluminium, given density of $l\ =\ 2700\ kg\ m^{-3}$. **[GU-2022]** Ans.: $1.56\ \times\ 10^2\ m^{-1}, 5.78\ \times\ 10^{-2}\ m^2\ kg^{-1}$

12. Given that the range, in standard air, of alpha particles from radium $\left(T_{1/2} = 1622 \text{ years}\right)$ is 3.36 cm, whereas from polonium $\left(T_{1/2} = 138 \text{ days}\right)$ this range is 3.85 cm. Calculate the half life of RaC for which the alpha particle range is 6.97 cm. [**GU-2022**] Ans.: $4.33 \times 10^{-10}\ s$

13. Calculate the kinetic energy of alpha particle in the following decay:
$Pu^{239} \rightarrow U^{235} + He^{4}$
Given, $M\ (Pu^{239}) = 239.052158\ MeV$, $M\ (U^{235}) = 235.043925\ MeV$,
$M\ (He^{4}) = 4.002603\ MeV$ [**GU-2022**] Ans.: 5.54 keV

14. For an atomic nucleus with atomic number Z and mass number A, which of the following is (are) correct? (a) Nuclear matter and nuclear charge are distributed identically in the nuclear volume. (b) Nuclei with $Z > 83$ and $A > 209$ emit α −radiations. (c) The surface contribution to the binding energy is proportional to $A^{2/3}$. (d) β −decay occurs when the proton to neutron ratio is large, but not when it is small.
[**JAM-2017**] Ans.: (a), (b), (c)

15. $^{210}_{84}Po$ decays to $^{206}_{82}Pb$ by α-emission and releases 5.04 MeV of energy. What percentage of reaction energy is carried away by the α- particle? If the masses of the Po-210 nucleus and the α-particle are respectively 209.9829 u and 4.0026 u then find the mass of the Pb-206. Ans.: 98%, 205.976 u

16. $^{226}_{88}Ra$ disintegrates emitting α-particle of energy 4.78 MeV. Find the recoil energy of the daughter nucleus. How many wavelengths of α-particle will fit inside the $^{226}_{88}Ra$ nucleus if the mass of the α-particle is 4.0026 u ? Take unit nuclear radius as $R_o = 1.4\ fm$. Ans.: 0.09 MeV, 2.5

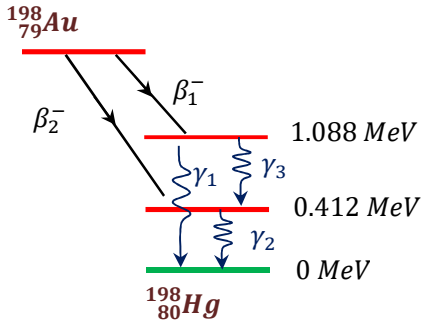
17. A stationary $^{200}_{82}Pb$ nucleus emits an α-particle with kinetic energy 5.77 MeV. Find the recoil velocity of the daughter nucleus. What percentage the total energy liberated in the decay is carried away by the daughter nucleus? Ans.: $3.5 \times 10^{5}\ m\ s^{-1}$, 2%

18. What is alpha disintegration energy? Calculate the kinetic energy of alpha particle in the following decay:

$$^{239}_{94}Pu \longrightarrow {}^{235}_{92}U + {}^{4}_{2}He$$

Given $M(^{239}_{94}Pu) = 239.052158\ amu, M(\ ^{235}_{92}U) = 235.043925\ amu, M(^{4}_{2}He) =$ 4.0002603 amu **[GU-2021]** Ans.: 5.15 MeV

19. Obtain the maximum kinetic energy of beta particle and the radiation frequencies of gamma rays in the decay scheme shown in the figure.



Given:

$m(^{198}_{79}Au) = 197.968233\ u$,

$m(^{198}_{80}Hg) = 197.966760\ u$, **[GU-2019]**

Ans.: $(K_\beta)_{max} = 1.371\ MeV, \gamma_1 = 2.63 \times 10^{20}\ Hz, \gamma_2 = 9.96 \times 10^{19}\ Hz, \gamma_3 = 1.63 \times 10^{20}\ Hz$,