UNIT 3: MATHEMATICS FOR AI

Very Easy Study Notes - Class XI Artificial Intelligence

Topics: Linear Algebra, Statistics, Probability and Calculus

This unit introduces the basic mathematical ideas used in Artificial Intelligence. The focus is on simple understanding, small examples and easy practice questions.

1. Why Do We Need Mathematics in AI?

Artificial Intelligence works mainly with data and numbers. Mathematics helps an AI system:

- represent data
- perform calculations
- find patterns
- compare values
- make predictions
- calculate uncertainty
- improve its predictions

Example: Suppose an AI system studies the relationship between hours studied and marks.

Hours Studied	Marks
1	40
2	50
3	60
4	70

In this simple example, mathematics helps the AI see that marks increase as study hours increase.

Main areas in this unit: Linear Algebra -> Statistics -> Probability -> Calculus.

2. Linear Algebra

Linear Algebra is the branch of mathematics that deals with scalars, vectors, matrices and related operations.

2.1 Scalar

A scalar is a single numerical value.

Examples: 5, 10, -3, 2.5

AI example: A learning rate such as 0.01 is a scalar.

2.2 Vector

A vector is an ordered collection of numbers.

Example: A = [2, 4, 6]

AI example: A student may be represented by the vector [17, 80, 90], where the values mean age, marks and attendance.

2.3 Vector Addition

Vectors of the same size are added by adding corresponding elements.

A = [2, 3], B = [4, 5]

A + B = [2+4, 3+5] = [6, 8]

2.4 Scalar Multiplication

A vector can be multiplied by a scalar.

A = [2, 3]

2A = [4, 6]

2.5 Matrix

A matrix is a rectangular arrangement of numbers in rows and columns.

A = [[1, 2], [3, 4]]

This matrix has 2 rows and 2 columns, so its order is 2 x 2.

B = [[1, 2, 3], [4, 5, 6]]

This matrix has 2 rows and 3 columns, so its order is 2 x 3.

2.6 Matrix Addition

Matrices of the same order are added by adding corresponding elements.

A = [[1, 2], [3, 4]]

B = [[2, 3], [4, 5]]

A + B = [[3, 5], [7, 9]]

2.7 Why Vectors and Matrices are Used in AI

AI systems deal with large amounts of numerical data. Images, text features and many other types of data can be represented numerically.

Example image matrix: [[0, 255, 255], [0, 0, 255], [255, 255, 0]]

In a simple black-and-white image, 0 may represent black and 255 may represent white.

3. Statistics

Statistics is the branch of mathematics concerned with collecting, organizing, analyzing and interpreting data.

Important basic ideas: Mean, Median, Mode and Range.

3.1 Mean

The mean is commonly called the average.

Mean = Sum of all observations / Number of observations

Example: Find the mean of 2, 4, 6, 8.

(2+4+6+8) / 4 = 20 / 4 = 5

Answer: Mean = 5.

3.2 Median

The median is the middle value when the data is arranged in order.

Example: 3, 5, 7, 9, 11 -> Median = 7.

If there are an even number of values, take the average of the two middle values.

For 2, 4, 6, 8: Median = (4+6)/2 = 5

3.3 Mode

The mode is the value that occurs most frequently.

Example: 2, 3, 3, 4, 5 -> Mode = 3.

3.4 Range

The range is the difference between the largest and smallest values.

Range = Largest value - Smallest value

Example: For 2, 5, 8, 10:

Range = 10 - 2 = 8

3.5 Combined Example

Consider the dataset: 2, 4, 4, 6, 9

Measure	Answer
Mean	5
Median	4
Mode	4
Range	7

3.6 Statistics in AI

Statistics helps AI understand datasets. For example, if five students score 60, 65, 70, 75 and 80, the mean score is 70. This gives a simple summary of the group.

4. Probability

Probability tells us how likely an event is to occur.

0 <= P(E) <= 1

- Probability 0 means impossible.
- Probability 1 means certain.
- A value between 0 and 1 means the event has some chance of occurring.

4.1 Basic Probability Formula

P(E) = Number of favourable outcomes / Total number of possible outcomes

4.2 Coin Toss Example

A coin has two outcomes: Head (H) and Tail (T).

$$P(H) = 1/2$$

$$P(T) = 1/2$$

4.3 Dice Example

A normal die has six outcomes: 1, 2, 3, 4, 5, 6.

Probability of getting 4:

$$P(4) = 1/6$$

Probability of getting an even number: favourable outcomes are 2, 4, 6.

$$P(\text{even}) = 3/6 = 1/2$$

Probability of getting an odd number: favourable outcomes are 1, 3, 5.

$$P(\text{odd}) = 3/6 = 1/2$$

4.4 Coloured Balls Example

A bag contains 3 red balls and 2 blue balls. Total balls = 5.

$$P(\text{Red}) = 3/5$$

$$P(\text{Blue}) = 2/5$$

4.5 Probability in AI

AI often makes decisions under uncertainty. For example:

Object	Probability
Cat	0.80
Dog	0.15
Rabbit	0.05

The AI may predict 'Cat' because it has the highest probability.

5. Calculus

Calculus is the branch of mathematics that studies change.

- Derivative: measures rate of change.
- Slope or gradient: shows how quickly a quantity is increasing or decreasing.

For this introductory unit, understanding the basic idea is more important than advanced calculations.

5.1 Simple Idea of Change

Time	Distance
1 hour	40 km
2 hours	80 km
3 hours	120 km

The distance increases by 40 km every hour. The rate of change is 40 km/hour.

5.2 Slope

Slope = Change in y / Change in x

m = (y2 - y1) / (x2 - x1)

Example: Find the slope between (1, 2) and (3, 6).

m = (6-2) / (3-1) = 4/2 = 2

Answer: Slope = 2.

5.3 Very Basic Derivative

If y = x^2, then its derivative is:

dy/dx = 2x

At x = 2:

2 x 2 = 4

So the rate of change at x = 2 is 4.

5.4 Why Calculus is Used in AI

An AI model makes errors while learning. Training tries to reduce the error.

Example: 20 -> 15 -> 10 -> 5

Calculus helps determine how model parameters should change to reduce error.

6. Quick Comparison

Area	Main Idea	Simple Example
Linear Algebra	Vectors and matrices	[2, 3, 4]
Statistics	Understanding data	Mean, median, mode
Probability	Chance of an event	P(Head) = 1/2
Calculus	Rate of change	Slope / derivative

7. Important Definitions for Examination

Linear Algebra: A branch of mathematics dealing with vectors, matrices and related operations.

Scalar: A single numerical value.

Vector: An ordered collection of numerical values.

Matrix: A rectangular arrangement of numbers in rows and columns.

Statistics: The branch of mathematics concerned with collecting, organizing, analyzing and interpreting data.

Mean: The sum of all observations divided by the total number of observations.

Median: The middle value of an ordered dataset.

Mode: The value that occurs most frequently in a dataset.

Probability: A numerical measure of the likelihood of an event occurring.

Calculus: The branch of mathematics concerned with change and rates of change.

Derivative: A quantity that represents the rate of change of one quantity with respect to another.

8. Solved Examples

Example 1 - Mean

Find the mean of 5, 10, 15, 20.

(5+10+15+20)/4 = 50/4 = 12.5

Answer: 12.5

Example 2 - Median

Find the median of 3, 5, 7, 9, 11.

Middle value = 7

Answer: 7

Example 3 - Mode

Find the mode of 2, 3, 3, 3, 5, 6.

3 occurs most frequently.

Answer: 3

Example 4 - Range

Find the range of 5, 8, 12, 15.

15 - 5 = 10

Answer: 10

Example 5 - Probability

A bag contains 4 red and 6 blue balls. Find P(Red).

P(Red) = 4/10 = 2/5

Answer: 2/5

Example 6 - Vector Addition

A = [2,4], B = [3,5]. Find A+B.

[2+3, 4+5] = [5,9]

Answer: [5,9]

9. Practice Questions - Very Short (1 Mark)

1. What is Linear Algebra?

2. What is a scalar?
3. Define a vector.
4. What is a matrix?
5. What is Statistics?
6. Define mean.
7. Define median.
8. What is mode?
9. Define probability.
10. What is the probability of a certain event?
11. What is the probability of an impossible event?
12. What is Calculus?
13. What does a derivative represent?
14. What is meant by the order of a matrix?
15. Give one use of mathematics in AI.

10. Practice Questions - Short (2 Marks)

16. Differentiate between a scalar and a vector with an example.
17. What is a matrix? Give an example of a 2 x 2 matrix.
18. What is the difference between mean and median?
19. Differentiate between median and mode.
20. Why is probability useful in AI?
21. What is the role of statistics in AI?
22. What is meant by rate of change?
23. Write any two uses of mathematics in Artificial Intelligence.
24. Write a vector representing the values 10, 20 and 30.
25. Find the order of the matrix [[1,2,3],[4,5,6]].

11. Numerical Practice

26. Find the mean of 4, 6, 8, 10, 12.
27. Find the median of 2, 5, 7, 8, 11.
28. Find the mode of 2, 4, 4, 5, 6, 4, 8.
29. Find the range of 5, 9, 12, 18, 20.
30. Find the mean, median and mode of 3, 5, 5, 7, 10.
31. If A = [1,2,3] and B = [4,5,6], find A+B.
32. Find 3[2,4].
33. Add the matrices [[1,2],[3,4]] and [[2,4],[1,3]].
34. A coin is tossed once. Find the probability of getting a Head.
35. A die is thrown once. Find the probability of getting: (a) 5 (b) an even number (c) an odd number.
36. A bag contains 5 red balls and 5 blue balls. Find the probability of selecting: (a) a red ball (b) a blue ball.
37. Find the slope between (1,3) and (3,7).

12. Answers to Numerical Practice

1. Mean = 8

2. Median = 7

3. Mode = 4

4. Range = 15

5. Mean = 6, Median = 5, Mode = 5

6. A+B = [5,7,9]

7. 3[2,4] = [6,12]

8. Result = [[3,6],[4,7]]

9. P(Head) = 1/2

10. P(5) = 1/6, P(even) = 1/2, P(odd) = 1/2

11. P(Red) = 1/2, P(Blue) = 1/2

12. Slope = (7-3)/(3-1) = 4/2 = 2

13. Important 5-Mark Questions

38. Explain the importance of mathematics in Artificial Intelligence. Briefly discuss Linear Algebra, Statistics, Probability and Calculus.
39. What is Statistics? Explain mean, median and mode with suitable examples.
40. What are scalar, vector and matrix? Explain each with an example.
41. What is Probability? Explain the basic probability formula with suitable examples.
42. What is Calculus? Explain the idea of rate of change and its importance in Artificial Intelligence.
43. Calculate the mean, median, mode and range of a given small dataset.

14. One-Page Memory Trick

L -> S -> P -> C

Linear Algebra -> Statistics -> Probability -> Calculus

Numbers arranged -> Data understood -> Chance measured -> Change measured