

2.4 Truth table method as a decision procedure

Decision procedure: what is it?

The aim of symbolic logic is two-fold:

- To symbolize the arguments and expressions.
- To determine the validity or invalidity of arguments.

In this section we shall deal with the latter aim. In logic of propositions there are many methods to test the validity of arguments and argument forms and also to examine whether a statement or statement forms are tautologous or not. These methods are called decision procedures. In other words, a mechanical procedure which will always give an answer for any validity problem is called a decision procedure for validity. Some standard decision procedures in propositional logic are:

1. Truth table method.
2. Shorter truth table method (reduction ad absurdum)
3. CNF(Conjunctive Normal Form)
4. DNF(Disjunctive Normal From)
5. Truth tree method.

Truth table is a decision procedure

Truth table method is one of the important decision procedures. The truth table method is a completely *mechanical* procedure. It requires no ingenuity or deep thought. It is like having an *algorithm* for the mechanical procedure described here, a set of rules that, if followed faithfully, will always yield the correct answer. This method is also known as **matrix method**.

Professor Virginia Klenk in her book *Understanding Symbolic Logic* points out three important properties of sentential logic that make possible the mechanical decision procedures.

- First, there are only a **finite number** of variables in each argument form.
- Second, sentential logic is a **two-valued logic**, which means that each variable can take only true or false substitution instances. Properties 1 and 2 together make it possible to list all possible combinations of truth values

for the variables – all possible (kinds of) substitution instances.

- Third, all our **operators** (logical constants) are truth functional. This means that once we have drawn up our list of possible truth values, we can compute the result for each possibility.

She points out that many systems lack these three important properties and therefore all systems of logic do not have mechanical procedure.

2.4.1 Truth table: what is it?

Truth table method is the most basic and perhaps the most popular decision procedure to examine the validity of arguments as well as **logical status of statements**. Truth tables are those which represent a systematic exhaustive enumeration of the possible truth values of combinations of simple propositions. A **truth table can be defined as a tabular representation in which all possible truth values of a compound statement is calculated out through the computations of the truth values of their simple constituent propositions**.

The truth table method has three basic functions:

1. Through the truth table method various operational definitions of logical constants can be given.
2. Truth table method enables us to decide how to determine the **validity or invalidity of arguments** in propositional logic. They also provide a mechanism whereby the truth values of **complicated expressions** can be worked out.
3. By means of the truth table method whether a proposition is **true or false** can be established. In addition to this, a truth table can conclusively establish whether a proposition is **tautologous** or **contradictory** or **contingent**.

Let us take an example of a truth table. The following is a truth table example for a conjunctive function ' $p \cdot q$ '.

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

In this truth table we can see that the value of ' $p \cdot q$ ' depends on the value of its component variables ' p ' and ' q '. The truth table has systematically represented all the possible truth values of combinations of simple propositions. The logical dependence between the compound expression and its constituent propositions has been clearly represented by this truth table.

The modern form of truth table was popularized by Emil Post (1897-1954) and Ludwig Wittgenstein (1889-1951) in the 1920s.

2.4.2 How to construct a truth table?

Construction of a truth table requires certain steps to follow. These steps are the following : The first step to construct a truth table is to symbolize the statement given. Symbolization determines the number of constituent propositions in a compound proposition. The constituent propositions are symbolized by taking the help of variables and the components are symbolized by logical constants. For example, 'if you go to college, you will learn'. In order to construct truth table of this statement we have to symbolize it in the form of ' $p \supset q$ '.

Secondly, we have to put truth values 'T' and 'F' under the respective columns. Because of the use of columns we can compute and display all possible truth values of the truth functional compound statements. For example, putting truth values 'T' and 'F' to the example given above, i.e. ' $p \supset q$ ', we

may construct a truth table for it in the following way.

p	$\supset$	q
T	T	T
T	F	F
F	T	T
F	T	T

Here, one point is to be kept in mind that for putting truth values for the logical constant ' $\supset$ ', we have to follow the rules of five basic truth functions which are discussed earlier.

Thirdly, before putting truth values 'T' or 'F', we have to check the number of variables by means of which the entire compound proposition is constituted. The number of columns will depend on the number of variables or repeated variables and on the number of logical constants. 'Variables + number of logical constants = Total number of columns' is the principle to determine the number of columns. Finally, we can say truth function is a function of logical components. In the example given above we have got ' $2+1 = 3$ ' number of columns.

Fourthly, we have to determine the number of rows, which is considered to be the most important step in constructing a truth table. If the number of variable is only one, then we have to put truth values 'T' and 'F' by drawing two rows. If the number of variable is two, then there will be four rows in which for one of the variables we have to put truth values TTFF and for the other TTF. **The formula which is adopted for counting the number of rows is '2ⁿ' where '2' is taken for two truth values of 'T' and 'F' (in two valued logic, a variable can have only two truth values, i.e. either 'T' or 'F') and 'n' represents the number of variable or variables.** By following this formula for one variable there will be two rows, for two variables there will be $2 \times 2 = 4$, for three there will be $2 \times 2 \times 2 = 8$, for four will be $2 \times 2 \times 2 \times 2 = 16$, and so on and so forth. The importance of rows lies in the fact that rows are supposed to provide all possible truth value situations for a given set of statements. In the example given above, the number of rows is four for two variables ' p ' and ' q '.

Fifthly, it has already been mentioned that a truth function is determined and finalized by the truth values of its components. For a compound statement, truth value is determined by the final logical constant. If all the truth values of final logical constants are 'T', then the compound proposition is a **tautological proposition**; if truth value 'F' occurs under the final logical constant, then the compound proposition is a **contradictory proposition**; if truth values 'T' and 'F' occur under the final logical constant, then the compound proposition is a **contingent proposition**. The example given below will clarify this point. On the other hand, with regard to an argument, we determine the validity or invalidity of it on the basis of the truth values of the premise and conclusion in some specific lines. It will be discussed in the next section.

Sixthly, understanding the use of bracket is important for computing and determining the final logical constant, the value of which determines the value of the entire compound proposition. Following mathematical use of bracket, in constructing a truth table for a compound proposition, first bracket '(', second bracket '{', and third bracket '[' are used respectively on the priority basis for performing the job. In other words, we perform the job starting with the constant which has least scope and ending it with the widest scope.

Now let us take an example of a compound proposition and let us apply the strategies given above to construct a truth table.

$$(p \supset q) \equiv (\sim p \vee q)$$

The compound proposition in example is given in symbolic form. Therefore, no question arises in symbolizing the statement. According to the second strategy mentioned above, we have to put truth values for each of the columns. In this particular example, there are two variables each of which occurs twice. Again, there are four logical constants in the statement. According to our strategy, there will be $4 + 4 = 8$ columns for four logical constants and four variables (here two variables occurs twice). As there are two variables (p and q), by following the '2ⁿ' principle, there will be four rows. Now, we have to compute the truth function in each parenthesis taking the truth values from left to

right. As there are two sets of first brackets, the truth value of those two will be computed to find out the result of the final logical constant, which is '≡' in this example. Now let us construct the truth table for the given compound proposition.

(	p	⊃	q	)	≡	(	~	p	∨	q	)
T	T	T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	T	F	T	F	F	F
F	T	T	T	T	T	F	T	T	T	T	T
F	T	F	T	T	T	F	T	T	T	T	T

Here we have seen that for the first set of bracket the given '(p ⊃ q)' we have assigned the truth value TTFF for 'p' and TTF for 'q'. The computed result in the set, i.e. under the logical constant, 'implication', is TFFT here. The same process is done with the second set of bracket and the computed result in the set, i.e. under the logical constant, 'vel' is TFFT. The main constant is computed by reconciling the two sets of results and the result under the final logical constant '≡' is TTTT. Therefore it is a tautological truth function.

2.4.3 Using Truth Tables to determine Tautology, Contradiction and Contingent Statements or Determination of logical status of a proposition, a propositional form, statement, statement form

A truth table method can be used to determine the logical status of a single, individual statement form or statement simply by checking its truth value in the final column. Klenk says, "Certain forms, for instance, have an interesting property: they can never, under any circumstances, turn out to be false, and this is something that can be determined by the truth table. Other forms have just the opposite property: they can never, in any case, turn out to be true. The former sort of statement forms, which can never be false, are called *tautologies*, and the latter,

which can never be true, are called, sensibly enough, **contradictions**. The intermediate cases, forms that are true in some instances and false in others, we will call **contingencies**.

Tautology

A statement or a statement form is considered to be a **tautology** if its truth table has only T-s in the final column. Tautologous propositions are always independent of experience, because if we try to convert them into any sort of contradictory form by other proofs, the attempt becomes only a futile attempt. Let us take an example:

Either I shall or shall not meet you tomorrow is a tautologous proposition, because the truth of this proposition can be known without any experience. This proposition can be symbolized as ' $p \vee \sim p$ '. Let us construct a truth table for this proposition:

p	$\vee$	$\sim p$	
T	T	F	T
F	T	T	F

In the above truth table the main logical constant is ' $\vee$ '. Under this column, we have got the truth value 'T' in both the cases. Therefore, the said proposition is a tautologous proposition.

Contradictory

A statement or a statement form is considered to be **contradictory** if its truth table has only F-s in the final column. The falsity of a contradictory proposition is, like a tautologous proposition, independent of experience. For example, *Bertrand Russell is or is not a philosopher*. This proposition can be symbolized as ' $p \cdot \sim p$ '. Let us construct a truth table for this proposition:

p	$\cdot$	$\sim p$	
T	F	F	T
F	F	T	F

In the above truth table the main logical constant is ' $\cdot$ ' (Dot). Under this column, we have got the truth value 'F' in both the cases. Therefore, the said proposition is a contradictory proposition.

Contingent

A statement or a statement form is considered to be **contingent** if its truth table has both T-s and F-s in the final column. The truth or falsity of a contingent proposition depends on experience. Therefore, it is not necessarily true. For example, *He likes tea or coffee*. This proposition can be symbolically represented in the following form: ' $p \vee q$ '. Let us construct a truth table for this statement.

p	$\vee$	q	
T	T	T	
T	T	F	
F	T	T	
F	F	F	

In the final column of the above truth table, we have seen below the final column a mixture of 'T-s' and 'F-s' occur. Therefore this proposition is regarded to be a contingent statement.

2.4.4 Employment of Truth Tables to determine the Truth Values of propositions

Truth tables are employed to determine the truth value of a proposition or a statement. A few examples are given below for better understanding of the point:

1. $(p \vee \sim p) \supset \sim p$

p	(p	$\vee$	$\sim$	p)	$\supset$	$\sim$	p
T	T	T	F	T	F	F	T
F	F	T	T	F	T	T	F
	1	4	3	2	7	6	5

In this truth table, for only one variable, we have drawn two rows. The final column in the truth table is column 7th, which has the truth values both T and F, as shown by the table. Therefore, the compound expression is **false** and it is a contingent proposition.

$$2. \sim(p \vee \sim q) \cdot (\sim q \supset p)$$

$$\sim (p \vee \sim q) \cdot (\sim q \supset p)$$

F	T	T	F	T	F	F	T	T	T
F	T	T	T	F	F	T	F	T	T
T	F	F	F	T	T	F	T	T	F
F	F	T	T	F	F	T	F	F	F
5	1	4	3	2	10	7	6	9	8

In the above truth table, we have seen that, for two variables p and q , there are four rows. Here, below the final logical constant there is a mixture of both T and F. Hence, the statement is false and it is a contingent expression.

$$3. (p \vee q) \equiv \sim(\sim p \supset q)$$

$$(p \vee q) \equiv \sim(\sim p \supset q)$$

T	T	T	F	F	F	T	T	T
T	T	F	F	F	F	T	T	F
F	T	T	F	F	T	F	T	T
F	F	F	F	T	T	F	F	F
1	3	2	9	8	5	4	7	6

In the above truth table, we have seen that, for two variables p and q , there are four rows. Here, below the final logical constant (column 9) there is the truth value of F only. Hence, the statement is false and it is a contradictory expression.

$$4. ((p \supset q) \cdot \sim q) \supset \sim p$$

$$((p \supset q) \cdot \sim q) \supset \sim p$$

T	T	T	F	F	T	T	F	T
T	F	F	T	T	F	T	F	T
F	T	T	F	F	T	T	T	F
F	T	F	T	T	F	T	T	F
1	3	2	6	5	4	9	8	7

In the above truth table, we have seen that, for two variables p and q , there are four rows. Here, below the final logical constant (column 9) there is the truth value of T only. Hence, the statement is true and it is a tautologous expression.

$$5. [(\sim p \vee \sim q) \supset q] \equiv r$$

$$[(\sim p \vee \sim q) \supset q] \equiv r$$

F	T	F	F	T	T	T	T	T
F	T	F	F	T	T	T	F	F
F	T	T	T	F	T	F	T	T
F	T	T	T	F	T	F	F	F
T	F	T	F	T	T	T	T	T
T	F	T	F	T	T	T	F	F
T	F	T	T	F	T	F	T	T
T	F	T	T	F	T	F	F	F
2	1	5	4	3	7	6	9	8

In the above truth table, we have seen that, for three variables p, q and r , there are eight rows. Here, below the final logical constant there is a mixture of both T and F. Hence, the statement is false and it is a contingent expression.